

# Modified Chaplygin Traversable Wormholes

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(Dated: February 1, 2008)

The modified Chaplygin gas (MCG) is a strong candidate for the unified model of dark matter and dark energy. The equation of state of this modified model is valid from the radiation era to the  $\Lambda$ CDM model. In early epoch (when  $\rho$  was large), dark matter had the dominant role while at later stages (when  $\rho$  is small), the MCG model behaves as dark energy. In this work, we have found exact solution of static spherically symmetric Einstein equations describing a wormhole for an inhomogeneous distribution of modified Chaplygin gas. For existence of wormhole solution, there are some restrictions relating the parameters in the equation of state for MCG and the throat radius of the wormhole. Physical properties and characteristics of these modified Chaplygin wormholes are analyzed in details.

PACS numbers: 04.20.Gz, 04.20.Jb, 98.80.Es

## I. INTRODUCTION

Recent observational evidences of anisotropy in the Cosmic Microwave Background radiation [1] and the data from type Ia Supernovae SN 1997H redshift [2-4] suggest that the universe is flat and is undergoing at present an accelerating phase preceded by a period of deceleration. These observational facts can not be explained by the known ordinary (baryonic) matter or radiation. There should be a significant amount of energy density of the universe which should be an extraordinary non-baryonic matter (dark matter) and energy (dark energy). Also, observational facts indicate that, dark matter should be of the order of 25% of the critical density and dark energy contributes about  $\frac{2}{3}$  of the the critical density [4-7]. Further, the universe in very recent past (at redshift  $z \lesssim 1$ ), is dominated by uniformly distributed dark energy with negative pressure and is probably responsible for the present phase of acceleration of the universe. In last few years, several models have been suggested to incorporate the recent observational evidences and among them, a single component perfect fluid having exotic equation of state, known as Chaplygin gas [8-18] is of great interest. A further generalized form is known as modified Chaplygin gas [the equation of state is given in Section III, equation (12)]. The parameters  $A$ ,  $B$  and  $\alpha$  are universal positive constants. The special choice  $A = 0$  and  $\alpha = 1$  corresponds to Chaplygin gas, first introduced to describe lifting forces on a plane wing in aerodynamics process [12]. Its generalization with the choice  $A = 0$ ,  $\alpha > 0$  is known as generalized Chaplygin gas (GCG), first introduced by Kamenshchik et al.[13] and Bento et al.[14]. For small energy density, the GCG model will have negative pressure while at high energy density, the model behaves as an almost pressureless fluid.

However, MCG model, an extension of GCG model [8, 15], can interpolate states between standard fluids at high density and constant negative pressure at low energy. In fact, the MCG model for  $\gamma = \frac{1}{3}$  describes the evolution from radiation epoch at early time to  $\Lambda$ CDM era at late time (where the fluid has constant energy density and behaves as a cosmological constant). This causes the present accelerating phase of the universe [8, 9, 15]. Another point that also goes against the GCG model is the measurement of the velocity of sound. In GCG model, the velocity of sound is negligible at early times and approaches the speed of light at late times. But in cosmic evolution, sound velocity comparable to the light velocity is not compatible.

From phenomenological view point, MCG model is interesting and can be motivated by the brane world interpretation [14]. Further, this model is consistent with various classes of cosmological tests viz. gravitational lensing [17], gamma-ray bursts [18] as well as the above mentioned observations. Moreover, the present MCG model (also GCG model) is naturally constrained through cosmological observables [16]. Also, for low energy density, the present model has similarity with GCG model and so the equation of state is that of a polytropic gas [15] with negative index. Therefore, it is possible to have astrophysical implications of the present model with an alternative way of restricting the parameters [16].

As in MCG model, though the pressure is initially positive, but with the evolution of the universe, the pressure becomes negative and finally behaves as phantom energy. It is well known that phantom energy violates null energy condition, a natural scenario for the existence of traversable wormholes [19, 20]. This purely theoretical ingredient possesses a peculiar property viz. "exotic matter", whose stress-energy tensor violates the null energy condition. This common fact of violation of null

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energy condition motivates to study wormholes supported by MCG equation of state. In this work, we shall construct static and spherically symmetric traversable wormhole geometries, satisfying the MCG equation of state. We shall consider matching of these wormhole geometries to an exterior vacuum space time and examine physical properties and characteristics of these solutions. We shall also study the traversability conditions [19, 21] and measure the amount of averaged null energy condition violating matter [22, 23] in particular cases.

The remainder of our paper is organized as follows: Section II provides basic equations for wormhole geometry. In Section III, we have formulated the fundamental equations describing wormhole in modified Chaplygin gas and the restrictions on the parameters of the equation of state of MCG have been presented. Section IV deals with asymptotic flatness of the wormhole geometry. The non-flat (aymptotically) wormhole solutions are bounded by an exterior vacuum space time for which, details of surface stresses are shown. In section V, some specific wormhole solutions are derived and their physical properties and characteristics are explored. We have considered the choices viz. (i) a particular relation between  $\Phi$  and  $b$  (in differential form), (ii) constant redshift function, (iii) two typical choices of the shape function and lastly (iv) isotropic pressure with a typical form of  $b(r)$ . The traversability condition has been studied in details for constant redshift function with a toy example. Also for the linear form of  $b(r)$ , the amount of averaged null energy condition violating matter has been estimated using "volume integral quantifier". Finally, the paper ends with discussion and concluding remarks in Section VI.

## II. BASIC EQUATIONS FOR WORMHOLE GEOMETRY

A spherically symmetric and static wormhole is given in Schwarzschild coordinates by the space time metric

$$ds^2 = -exp [2\Phi(r)] dt^2 + \frac{dr^2}{1 - \frac{b(r)}{r}} + r^2 d\Omega^2 \quad (1)$$

where the two arbitrary functions of  $r$  viz.  $\Phi(r)$  and  $b(r)$  are as usual termed as the redshift function and the shape function respectively. The radial coordinate  $r$  ranges over  $[r_0, \infty)$  where the minimum value  $r_0$  corresponds to the radius of the throat of the wormhole. But there may be a cut-off of the stress-energy tensor at any finite junction radius  $R$ , where the interior space time matches to an exterior

vacuum solution.

The following properties need to be imposed for the existence of a wormhole solution: [19, 20]

$$\text{I. As } r_0 \text{ is the throat radius, so } b(r_0) = r_0 \quad (2)$$

$$\text{II. A flaring out condition of the throat, i.e}$$

$$\frac{(b - b'r)}{b^2} > 0, \text{ which at the throat simplifies to } b'(r_0) < 1 \quad (3)$$

$$\text{III.}$$

$$b(r) < r \text{ for } r > r_0 \quad (4)$$

(regularity of the metric coefficient)

IV. Traversability criteria: For traversable wormhole, there should not be any horizons present i.e,  $\Phi$  must be finite everywhere.

Using the Einstein field equation  $G_{\mu\nu} = 8\pi T_{\mu\nu}$ , in an orthonormal reference frame ( $G = C = 1$ ), we obtain the following set of equations

$$\rho(r) = \frac{b'}{8\pi r^2} \quad (5)$$

$$p_r(r) = \frac{1}{8\pi} \left[ \frac{2}{r} \left( 1 - \frac{b(r)}{r} \right) \Phi' - \frac{b}{r^3} \right] \quad (6)$$

and

$$p_t(r) = \frac{1}{8\pi} \left( 1 - \frac{b(r)}{r} \right) \left[ \Phi'' + \Phi' \left( \Phi' + \frac{1}{r} \right) \right] - \frac{(b'r - b)}{2r^2} \left( \Phi' + \frac{1}{r} \right) \quad (7)$$

in which  $\rho(r)$ ,  $p_r(r)$  and  $p_t(r)$  are the energy density, radial pressure and tangential pressure respectively. The energy conservation equation gives

$$p'_r = \frac{2}{r} (p_t - p_r) - (\rho + p_r) \Phi' \quad (8)$$

This equation can be interpreted as the hydrostatic equilibrium equation for the material threading the wormhole.

The above Einstein equations can be rearranged to obtain metric coefficients in terms of the components of the stress-energy tensors as

$$b' = 8\pi\rho(r)r^2 \quad (9)$$

$$\Phi' = \frac{b + 8\pi p_r r^3}{2r^2 \left(1 - \frac{b(r)}{r}\right)} \quad (10)$$

The above flaring out condition implies that the wormhole should be threaded with matter violating the null energy condition (NEC) (a fundamental property of wormholes). Matter that violates the NEC is termed as exotic matter [19, 24, 25]. The mathematical form of the null energy condition is  $T_{\mu\nu}K^\mu K^\nu \geq 0$  for any null vector  $K^\mu$ . However, in an orthonormal reference frame, we may choose  $K^\mu = (1, 1, 0, 0)$  so that the violation of NEC simplifies to  $\rho + p_r < 0$  i.e,

$$\frac{1}{8\pi} \left[ \frac{b'r - b}{r^3} + 2 \left(1 - \frac{b}{r}\right) \frac{\Phi'}{r} \right] < 0 \quad (11)$$

Note that, if we evaluate at the throat considering the flaring out condition and the finite character of  $\Phi(r)$ , the above inequality (11) is automatically satisfied, i.e, the null energy condition is violated.

### III. MODIFIED CHAPLYGIN GAS: THE EQUATION OF STATE

The MCG is described by the equation of state [8, 9]

$$p_r = A\rho - \frac{B}{\rho^\alpha}, \quad 0 < \alpha \leq 1 \quad (12)$$

where the parameters  $A$  and  $B$  are two universal positive constants. This equation of state shows a radiation era (when  $A = 1/3$ ) at one extreme (when the matter density  $\rho$  is very large) and a  $\Lambda$ CDM model at the other extreme (when  $\rho$  is infinitesimally small). However, at all stages it shows a mixture. Further, somewhere in between the two extreme, there is one stage when the pressure vanishes and the matter content is equivalent to pure dust (this cosmological model can be considered from the field theoretical point of view by introducing a scalar field having self-interacting potential [9]).

Now eliminating  $\rho$  and  $p_r$  among the equations (9), (10) and (12), we have the relation between  $\Phi$  and  $b$  (in differential form) as

$$\Phi' = \frac{\left[ b + Arb' - \frac{rB(8\pi r^2)^{\alpha+1}}{(b')^\alpha} \right]}{2r^2 \left(1 - \frac{b}{r}\right)} \quad (13)$$

Any solution of the metric (1) satisfying relation (13) is termed as modified Chaplygin wormhole provided it obeys the restrictions (mentioned in the previous section) for wormholes. In principle, we now have four equations (8)-(10), (13) containing five unknowns viz. two metric coefficients  $\Phi(r)$  and  $b(r)$  and three physical quantities (components of stress-energy tensor)  $\rho$ ,  $p_r$  and  $p_t$ . So we may restrict one of the above unknown quantity (or a relation among them) for a specific solution. In the present problem, we shall choose  $b(r)$  or  $\Phi(r)$  so that the solution fulfills the criteria of wormholes. Note that, as we shall consider the matter density  $\rho$  to be positive, so from equation (5), the shape function  $b(r)$  should be such that  $b'(r) > 0 \forall r$ .

Now at the throat  $r = r_0$ , the components of the stress-energy tensor are (for the choice  $\alpha = 1$ )

$$\left. \begin{aligned} \rho_0 &= \rho(r = r_0) = \frac{1}{2A} \left[ \sqrt{\left(\frac{1}{8\pi r_0^2}\right)^2 + 4AB} - \frac{1}{8\pi r_0^2} \right] \\ p_{r(0)} &= -\frac{1}{8\pi r_0^2} \\ p_{t(0)} &= \frac{1-b'(r_0)}{2r_0} \left( \Phi'_0 + \frac{1}{r_0} \right) \end{aligned} \right\} \quad (14)$$

The restriction  $b'(r_0) < 1$  for the wormhole geometry restricts the parameter  $A$  and  $B$  in MCG equation of state as

$$B < \frac{(1+A)}{(8\pi r_0^2)^2} \quad (15)$$

In the present context, the violation of null energy condition i.e,  $\rho + p_r < 0$  demands the energy density to be restricted as

$$\rho^{\alpha+1} < \frac{B}{1+A} < \left( \frac{1}{8\pi r_0^2} \right)^2 \quad (16)$$

This is not unusual in MCG cosmology models. Usually in FRW cosmology, the evolution of  $\rho$  is given by (see eqn. (8) in Ref. [9])

$$\rho = \left[ \frac{B}{1+A} + \frac{C}{a^{3(1+A)(1+\alpha)}} \right]^{(1/1+\alpha)} \quad (17)$$

where  $a$  is the scale factor in FRW model and  $C$  is arbitrary integration constant. So if  $C$  is chosen to

be positive, then  $\rho^{\alpha+1} > \frac{B}{1+A}$  i.e, dominant energy condition is satisfied. But for the choice of negative values of  $C$ , the restriction (16) is automatically satisfied, a fundamental ingredient in wormhole physics. Moreover, the expression for the velocity of sound is

$$v_s^2 = \frac{\partial p_r}{\partial \rho} = A + \frac{B\alpha}{\rho^{\alpha+1}} \quad (18)$$

The condition that  $v_s$  should not exceed the velocity of light demands

$$\rho > \left( \frac{B\alpha}{1-A} \right)^{(1/(1+\alpha))} \quad (19)$$

Thus combining (16) and (19) we have the constraint

$$\frac{B\alpha}{1-A} < \rho^{\alpha+1} < \frac{B}{1+A} < \frac{1}{64\pi^2 r_0^2} \quad (20)$$

It should be mentioned that in the domain of exotic matter,  $\frac{\partial p}{\partial \rho}$  may not be interpreted as the speed of sound due to the lack of knowledge of a microphysical model of exotic matter. On the other hand, there are examples of exotic behaviour such as Casimir effect and the false vacuum [26] where  $\frac{\partial p}{\partial \rho} < 0$ .

#### IV. ASYMPTOTIC FLATNESS AND STRESS-ENERGY TENSOR CUT-OFF

In the above, the basic structure of the solution has been shown and the restrictions for the parameters involved are presented. A solution for which  $\Phi \rightarrow 0$  and  $\frac{b}{r} \rightarrow 0$  as  $r \rightarrow \infty$ , corresponds to asymptotically flat space times. However, every solution may not be related to asymptotically flat space time. Then the stress-energy may be cut-off at a finite radial coordinate  $R$  and the interior solution of metric (1) is matched to an exterior vacuum space time at the junction interface  $r = R$ . The junction  $r = R$  may be a thin shell (if there are surface stresses at the junction surface) or simply a boundary surface (zero surface stresses). Hence for solutions corresponding to asymptotically non-flat geometry, the MCG distribution is restricted to the neighbourhood of the throat of the corresponding wormhole solution.

If the exterior vacuum space time is chosen for simplicity as the Schwarzschild space time i.e,

$$ds_E^2 = - \left( 1 - \frac{2M}{r} \right) dt^2 + \frac{dr^2}{\left( 1 - \frac{2M}{r} \right)} + r^2 d\Omega_2^2 \quad (21)$$

then one must have

$$R > r_b = 2M \text{ (event horizon).}$$

Using the junction conditions due to Darmois-Israel formalism [27], the components of the surface stresses of a dynamic thin shell [28, 29] are

$$\sigma \text{ (surface energy density)} = -\frac{1}{4\pi R} \left[ \sqrt{1 - \frac{2M}{R}} + \dot{R}^2 - \sqrt{1 - \frac{b(R)}{R}} + \dot{R}^2 \right] \quad (22)$$

and

$$\mathcal{P} \text{ (tangential surface pressure)} = \frac{1}{8\pi R} \left[ \frac{1 - \frac{M}{R} + \dot{R}^2 + R\ddot{R}}{\sqrt{1 - \frac{2M}{R}} + \dot{R}^2} - \frac{(1 + R\Phi')(1 - \frac{b(R)}{R} + \dot{R}^2) + R\ddot{R} - \frac{\dot{R}^2(b-b'R)}{2(R-b)}}{\sqrt{1 - \frac{b(R)}{R}} + \dot{R}^2} \right] \quad (23)$$

The above expressions for the components of stress-energy tensor can be simplified to a great extent for the static case [30] by imposing  $\dot{a} = 0 = \ddot{a}$ , where the overdot denotes derivative with respect to the proper time  $\tau$ .

Further, if  $M_s = 4\pi r^2 \sigma$  denotes the surface mass of the thin shell (for static case), then the total mass  $M_T$  of the wormhole has the expression [21]

$$M_T = \frac{b(R)}{2} + M_s \left[ \sqrt{1 - \frac{b(R)}{r}} - \frac{M_s}{2R} \right] \quad (24)$$

Also for the static case, if we have the boundary surface i.e,  $\sigma = 0 = \mathcal{P}$ , then one obtains the following relationships

$$b(R) = 2M \quad \text{and} \quad R = 2M \left( \frac{\zeta - 1/2}{\zeta - 1} \right) \quad (25)$$

As  $R > 2M$ , so the redshift parameter  $\zeta [= 1 + R\Phi'(R)]$  is restricted by  $\zeta > 1$ .

#### V. SPECIFIC WORMHOLE SOLUTION

##### A. A special relation between the redshift function and the shape function:

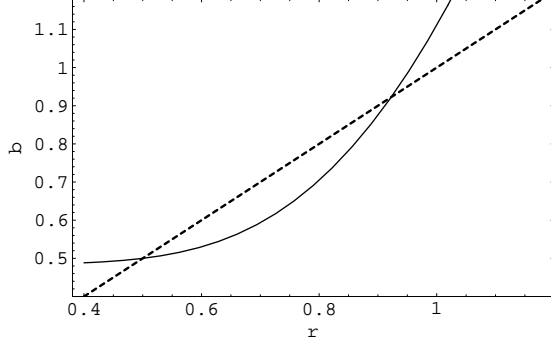


fig.(a) shows the variation of  $b(r)$  (given by eq.(27)) over the radial coordinate  $r$  for  $\mu = 1$  and  $r_0 = \frac{1}{2}$ .

Suppose we choose

$$\Phi' = \frac{A}{2} \frac{b'}{(r-b)} \quad \text{i.e.,} \quad \Phi = \int \frac{A}{2} \frac{b'}{(r-b)} dr \quad (26)$$

Then from equation (13), the solution for  $b$  becomes

$$b(r) = [r_0^2 + \mu(r^6 - r_0^6)]^{1/2}, \quad \mu = \frac{64\pi^2 B}{3} \quad (27)$$

with  $3\mu r_0^4 < 1$  for  $b'(r_0) < 1$ .

Clearly, this solution does not correspond to an asymptotically flat space time. So the MCG is confined in a neighbourhood of the throat and the above solution can be matched to exterior vacuum geometry (as described in Section IV). Further, the restriction  $b(r) < r$  yields a range of  $r$  (for which wormhole solution is possible) viz.

$$r_0 < r < \left[ \sqrt{\frac{4 - 3\mu r_0^4}{4\mu}} - \frac{r_0^2}{2} \right]^{1/2} \quad (28)$$

and is shown diagrammatically in fig.(a).

### B. Constant redshift function: $\Phi'(r) = 0$

This choice of the redshift function gives (from (13)) the differential equation in the shape function as

$$b(b')^\alpha + Ar(b')^{\alpha+1} - (8\pi)^{\alpha+1} Br^{2\alpha+3} = 0 \quad (29)$$

One can immediately inspect that  $b(r) = r_0 \left(\frac{r}{r_0}\right)^3$  is a particular solution for  $b(r)$  but it is not a wormhole solution as  $b'(r_0) < 1$  (flaring out condition) is

violated. However, for traversability criteria for constant  $\Phi$ , we need only  $b'(r)$  at the throat, which can be obtained from (29) as (for  $\alpha = 1$ )

$$b'(r_0) = \frac{\sqrt{1 + 12A\mu r_0^4} - 1}{2A} \quad (30)$$

Note that for the flaring out condition we must have

$$3\mu r_0^4 < 1 + A, \quad \mu = \frac{64\pi^2 B}{3} \quad (31)$$

We shall now examine an important concept of wormhole physics namely the traversability conditions for a human being to go forward through the wormhole. The basic criteria for a smooth journey is that the acceleration felt by the traveler should not exceed earth's gravity  $g_e$  (for details see ref.[19-21]) i.e.,

$$\left| \left(1 - \frac{b}{r}\right)^{1/2} e^{-\Phi} (\gamma e^\Phi)' \right| \leq g_e \quad (32)$$

Moreover, for smooth journey across the wormhole, the tidal acceleration also should not exceed the earth's gravitational acceleration ( $g_e$ ) and we have the following restrictions [19]

$$\left(1 - \frac{b}{r}\right) \left[ \Phi'' + \Phi'^2 - \frac{(b'r - b)}{2r(r-b)} \Phi' \right] |\eta^{\hat{i}'}| \leq g_e \quad (33)$$

$$\frac{\gamma^2}{2r^2} \left[ v^2 \left( b' - \frac{b}{r} \right) + 2(r-b)\Phi' \right] |\eta^{\hat{j}'}| \leq g_e \quad (34)$$

Here the separation between two arbitrary parts of the traveler's body is denoted by  $|\eta^{\hat{i}'}|$  and is chosen to be  $2m$  along any spatial direction in the traveler's reference frame as in the literature [21, 25].

Suppose a human being travels with velocity  $v$  along the radial direction such that  $v > 0$  in  $-l_1 < l < l_2$  but  $v = 0$  at  $l = -l_1$  and  $l_2$ , where the two space stations are assumed to be located (just outside the junction shell radius  $R$ ) at proper radial distances  $l = -l_1$  and  $l = l_2$  respectively. ( $l = \int \frac{dr}{\sqrt{1 - \frac{b}{r}}}$  is the proper radial distance). Then the time of travel across the wormhole is given by [19, 21, 31]

$$(\Delta T)_t = \int_{-l_1}^{l_2} \frac{dl}{v\gamma} \quad \text{and} \quad (\Delta T)_0 = \int_{-l_1}^{l_2} \frac{dl}{ve^\Phi} \quad (35)$$

where  $\gamma = [1 - v^2]^{-1/2}$  and  $(\Delta T)_t$  is the proper time interval of the journey recorded by the traveler's clock,  $(\Delta T)_0$  is the coordinate time interval recorded

by observers situated at the stations.

For simplicity, if we choose wormholes corresponding to  $\Phi = \text{constant}$  (i.e, constant redshift), then for a traveler moving with non-relativistic velocity (i.e,  $\gamma \approx 1$ ), the restrictions (32) and (33) are identically satisfied while the inequality (34), when evaluated at the throat, gives the upper limit of the velocity as

$$v \leq r_0 \sqrt{\frac{2g_e}{[1 - b'(r_0)] |\eta^{2'}|}} \quad (36)$$

As a toy example if we choose  $b'(r_0) = \frac{1}{2}$ ,  $r_0 = 10^2 \text{m}$ , then one obtains  $v \approx 4 \times 10^2 \text{ m/s}$  (choosing the equality sign) for the maximum traversal velocity. Further, choosing the junction radius  $R \simeq 10^4 \text{m}$ , the traversable time is  $(\Delta T)_t \simeq (\Delta T)_0 \simeq \frac{2a}{v} \approx 50 \text{ sec}$ .

### C. Specific choice for the shape function:

#### I. $b(r) = r_0 \left(\frac{r}{r_0}\right)^n$

The particular choice of the shape function

$$b(r) = r_0 \left(\frac{r}{r_0}\right)^n, \quad 0 < n < 1 \quad (37)$$

satisfies the wormhole criteria viz.

$$(i) \ b(r_0) = r_0, \quad (ii) \ b'(r) = n \left(\frac{r}{r_0}\right)^{n-1} \text{ i.e,}$$

$$b'(r_0) = n < 1, \quad (iii) \ 1 - \frac{b(r)}{r} = 1 - \left(\frac{r_0}{r}\right)^{1-n} < 1 \quad (38)$$

Also  $\frac{b(r)}{r} = \left(\frac{r_0}{r}\right)^{1-n} \rightarrow 0$  as  $r \rightarrow \infty$ .

Then solving equation (13) we have

$$\begin{aligned} \Phi(r) = \Phi_0 + \frac{(1 + An)}{2(1 - n)} \left[ \ln \left| \left(\frac{r}{r_0}\right)^{1-n} - 1 \right| - \ln \left| \frac{r}{r_0} \right|^{1-n} \right] \\ - \frac{32\pi^2 B r_0^4}{n(1 - n)} \left[ \ln \left| \left(\frac{r}{r_0}\right)^{1-n} - 1 \right| + \alpha z \right. \\ \left. + \frac{\alpha(\alpha - 1)}{2} \frac{z^2}{2} + \dots \dots \dots \right] \end{aligned} \quad (39)$$

where  $z = \left(\frac{r}{r_0}\right)^{1-n} - 1$ ,  $\alpha = \frac{5-n}{1-n}$  and  $\Phi_0$  is the integration constant. It can be seen easily that the above solution describes a non traversable wormhole

as there is an event horizon at the throat  $r = r_0$ . However, it is possible to make the wormhole solution traversable by imposing the restriction

$$\frac{1 + An}{2(1 - n)} = \frac{32\pi^2 B r_0^4}{n(1 - n)} \quad (40)$$

Then  $\Phi(r)$  simplifies to

$$\begin{aligned} \Phi(r) = \Phi_0 - \frac{1 + An}{2(1 - n)} \left[ \ln \left| \frac{r}{r_0} \right|^{1-n} + \alpha z \right. \\ \left. + \frac{\alpha(\alpha - 1)}{2} \frac{z^2}{2} + \dots \dots \dots \right] \end{aligned} \quad (41)$$

As  $\Phi(r)$  does not approach to a constant limit as  $r \rightarrow \infty$ , so the solution is not asymptotically flat. Hence we may match this interior solution to an exterior vacuum space time at a junction radius  $R$  and  $\Phi_0$  can be determined from the boundary conditions of  $\Phi$  at the junction interface.

#### II. $b(r) = r_0 + d(r - r_0)$

This typical choice of  $b(r)$  satisfies as above all the criteria to describe a wormhole provided  $0 < d < 1$ . It is clear that  $\frac{b}{r} \rightarrow 0$  as  $r \rightarrow \infty$  and the solution for  $\Phi$  becomes

$$\begin{aligned} \Phi(r) = \Phi_0 + \frac{(1 + Ad)}{2(1 - d)} \ln|r - r_0| - \frac{1}{2} \ln r - \frac{32\pi^2 B}{d(1 - d)} \\ \left[ r_0^4 \ln|r - r_0| + \frac{1}{4} (r - r_0)^4 + 4r_0^3(r - r_0) \right. \\ \left. + 3r_0^2(r - r_0)^2 + \frac{4}{3} r_0(r - r_0)^3 \right] \end{aligned} \quad (42)$$

which clearly describes a non-traversable wormhole. However, the choice

$$A = \frac{1}{d} [64\pi^2 B r_0^4 - 1]$$

makes  $\Phi$  finite at  $r = r_0$  and the solution describes a traversable wormhole (but not asymptotically flat). So the MCG is confined around the throat up to a junction interface  $r = R$ .

We shall now examine how much of matter is in the space time, which violates the averaged null energy

condition. According to Visser et al.[22, 23], this information can be obtained by "volume integral quantifier" defined as

$$I_v = \int [\rho(r) + p_r(r)] dV$$

$$= \int_{r_0}^R (r-b) \left[ \ln \left( \frac{e^{2\Phi}}{1 - \frac{b}{r}} \right) \right]' dr \quad (43)$$

Now using the solution (42) for  $\Phi$  and the choice  $b(r) = r_0 + d(r - r_0)$ , the above integral simplifies to

$$I_v = -(1+Ad) \left[ \frac{a^5}{5} + a^4 r_0 + 2a^3 r_0^2 + 2a^2 r_0^3 \right]$$

$$+ [1 + Ad - r_0^4(1-d)]a \quad (44)$$

where  $a = R - r_0$  is the radial distance of the junction interface from the throat. Thus as  $a \rightarrow 0$ ,  $I_v \rightarrow 0$ . This shows that, similar to the phantom [21] or GCG wormholes [25], it is possible to construct a wormhole solution in MCG, where the amount of modified Chaplygin gas is arbitrarily small. This conclusion is in agreement with the results in the literature [21, 25] regarding Chaplygin gas. The interesting point to be noted is that, theoretically it is possible to construct wormholes supported by infinitesimal amount of exotic fluids in cosmology, which may responsible for the present accelerating phase of the universe.

#### D. Isotropic Pressure:

In case of isotropic pressure (i.e,  $p_r = p_t$ ), the energy conservation equation (8) simplifies to

$$\Phi' = -\frac{(A\rho^{\alpha+1} + B\alpha)}{[(A+1)\rho^{\alpha+1} - B]\rho} \rho' \quad (45)$$

which on integration gives

$$\Phi = -\frac{A}{(A+1)(\alpha+1)} \ln |(A+1)\rho^{\alpha+1} - B|$$

$$+ \frac{\alpha}{\alpha+1} \ln \left| \frac{(A+1)\rho^{\alpha+1}}{(A+1)\rho^{\alpha+1} - B} \right| + c \quad (46)$$

The constant  $c$  can be determined from the fact that, at the throat  $r = r_0$ ,

$$\rho_0 = \frac{1}{2A} \left[ \sqrt{\left(\frac{1}{8\pi r_0^2}\right)^2 + 4AB} - \frac{1}{8\pi r_0^2} \right] \text{ [see eqn.(14)]}$$

and  $\phi(r = r_0) = \phi_0$  (say), i.e, (for  $\alpha = 1$ )

$$c = \phi_0 + \frac{A}{2(1+A)} \ln |(A+1)\rho_0^2 - B| - \frac{1}{2} \ln \left| \frac{(1+A)\rho_0^2}{(A+1)\rho_0^2 - B} \right|$$

Suppose we choose the shape function as [32]

$$b(r) = r_0 + \gamma^2 r_0 \left[ 1 - \frac{r_0}{r} \right] \quad (47)$$

with  $0 < \gamma^2 < 1$ .

Then from field equation (5) we have

$$\rho(r) = \frac{\rho_1}{r^4}, \quad \rho_1 = \frac{\gamma^2 r_0^2}{8\pi} \quad (48)$$

and the explicit expression for  $\Phi$  becomes

$$\Phi(r) = c - \frac{A}{(A+1)(\alpha+1)} \ln \left| \frac{(A+1)\rho_1^{\alpha+1}}{r^{4(\alpha+1)}} - B \right|$$

$$+ \frac{\alpha}{\alpha+1} \ln \left| \frac{(A+1)\rho_1^{\alpha+1}}{(A+1)\rho_1^{\alpha+1} - r^{4(\alpha+1)}B} \right| \quad (49)$$

Clearly, the solution is not asymptotically flat and the radius of the junction interface should be restricted by

$$R < \left[ \frac{(A+1)\rho_1^{\alpha+1}}{B} \right]^{1/4(\alpha+1)} \quad (50)$$

Finally, it should be mentioned that wormhole solution for constant energy density is identical to that in ref. [25], so we have not presented it here.

## VI. DISCUSSION AND CONCLUDING REMARKS

The modified Chaplygin gas model is a unified model of dark matter and dark energy, which causes the present accelerated expansion of the universe. First of all in the context of cosmology, the MCG model describes a smooth transition from a decelerated expansion to the present epoch of cosmic acceleration (i.e, a continuous transition from radiation era to  $\Lambda$ CDM model). Also from the phenomenological view point, the model describes unified macroscopic prescription of dark matter and dark energy. Lastly, the model is favorable than the GCG model (clearly presented in the introduction) and is consistent with several cosmological tests.

In the present work, we have examined whether wormhole solutions (traversable or non-traversable) are possible in the background of MCG model. It is

found that, for the existence of a generic solution of a wormhole, the parameters in the equation of state of MCG should be restricted as (for  $\alpha = 1$ )

$$\frac{B}{1+A} < (8\pi r_0^2)^2$$

and consequently, there is violation of null energy condition (a fundamental requirement in wormhole physics). We have presented four specific wormhole solutions and analyzed the physical properties with characterization of these modified Chaplygin wormholes. In the first solution, we have assumed a differential relation between the redshift function  $\Phi(r)$  and the shape function  $b(r)$ . The wormhole solution is valid within a finite range of  $r$  which is matched with an exterior vacuum space time. The range of  $r$  for the wormhole solution is shown graphically. For the choice of constant redshift function, no analytic form of the shape function is possible. Only we have determined the restriction in the parameters involved to satisfy the traversability criteria. Then solutions are obtained for two specific choices of the shape function. In both cases, the wormhole solutions can be made traversable by imposing certain restriction on the parameters involved. For the second choice, we have calculated 'volume integral quantifier' ( $I_v$ ) to have an idea about the amount of matter violating averaged null energy condition. We have seen that  $I_v \rightarrow 0$  as the junction interface approaches to the

throat. Lastly, we have obtained wormhole solution for isotropic pressure. Here also due to complicated form of  $\Phi$  a function of energy density  $\rho$ , the form of the shape function is assumed and solution is confined in a finite restricted region. Finally, we have shown in general the traversability condition for a human being. The restrictions are simplified a great deal for constant redshift. We have finally estimated the time of travel for a traveler moving with non-relativistic speed. It should be mentioned that the space stations should be located at large  $r$ , so that the geometry at the stations is nearly flat [31]. As a concluding remark, we may say that the MCG traversable wormholes have far-reaching physical and cosmological implications. In addition, for the use of interstellar shortcuts, an absurdly advanced civilization may use them as time machines (probably violating causality) [20, 33, 34] (for review see [35]).

#### Acknowledgement:

The majority of the work has been done during a visit to IUCAA under the associateship programme. The authors gratefully acknowledge the warm hospitality and facilities of work at IUCAA. Also T.B is thankful to CSIR, Govt. of India, for awarding Junior Research Fellowship and to Anusua Baveja for fruitful discussions.

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